ORIGINAL ARTICLE Received 11 August. 2017 Accepted 30 May. 2018

Vol. 7, Issue 3, 98-105, 2018

Academic Journal of Psychological Studies

ISSN: 2333-0821

ajps.worldofresearches.com

Mathematical Problem Posing and Problem Solving: A Case of Turkish Students

Elif Esra Arikan[*1], Hasan Unal[2]

1. Faculty of Science and Letters, Yildiz Technical University, Istanbul, 34220, Turkey.
2. Faculty of Education, Yildiz Technical University, Istanbul, 34220, Turkey.

A B S T R A C T

The purpose of this study was to compare the mathematical problem posing and problem solving of 133 seventh grade students and 117 eighth grade student in Turkey. The study consisted of problem posing question according to figural pattern and problem solving question with regard to making sense which is division-with-remainder (DWR). The questions were selected CAI's study. Data were analyzed both of qualitative (content analysis) and quantitative (independent T test). This study was non-experimental comparative study in nature. As a result, Turkish students come behind of in international arena.

Keywords: Problem Posing, Problem Solving, Making Sense.

INTRODUCTION

In contemporary mathematics education, problem solving and problem posing activity are seen as cornerstones of students' experiences in the classes. Since the end of the previous century, many researchers has explored problem solving and problem posing. The main investigations of problem solving and problem posing were that how they affect the development of mathematical understanding. Primary level students' reasoning abilities are improved by doing problem posing activity(Mathematics, 2000; Silver, 1994; Singer, Ellerton, & Cai, 2015).

Some researchers as Lai, Zhu, Chen, and Li (2015), Watson and Mason (2002), Silver and Cai (1996), Ellerton (1986) and Krutetskiĭ, WIRSZUP, and Kilpatrick (1976) has found a relationship between problem solving and problem posing. Under these study, it was emphasized that the more ability students in problem solving are the more ability in problem posing. Also, problem posing and problem solving has been proposed as a way for students to unify both mathematics and real life. Besides, they has been claimed useful for autonomy learning and creativity mathematical thinking.

Australia, America, China, Singapore, Japanese and the other countries which are top scorers in international studies such as PISA and TIMSS, problem solving has been taken a place in their curricula. Moreover, according to syllabuses of these countries, a student can pose own problem as the student is able to solve a problem. In Turkey, problem posing has been in mathematics curriculum from 2006 by developing new mathematics curriculum(Kilic, 2013). Turkish mathematics textbooks have reorganized in concordance with the curriculum. Namely,

[*] . Corresponding Author: arikanee@gmail.com

DOI: In prossing

To cite this article: Arikan, E. E., Unal, H. (2018). Mathematical Problem Posing and Problem Solving: A Case of Turkish Students. *Academic Journal of Psychological Studies,* 7 (3), 98-105.

Turkish students, who are from 1 to 8 grades, mingle with problem posing activity from revision in mathematics education.

Problem of Research

The purpose was to examine 7th and 8th grade students' realistic approach to DWR (division with remainder) problems and the generative aspects of their mathematical thinking. Purpose of this study, two tasks was selected:

a) Problem posing task according to figural pattern situation

b) Story problem which is DWR problem.

The study was originated from Silver and Cai (1996) study. He compared US and Chinese students in terms of mathematical performance. The cross-national study was to compare with each other on four tasks that a) four computational exercises, b) a problem posing task based on a given figural pattern situation, c) division with remainder (DWR) story problem and d) a figural pattern problem. In our study, we would like to know what our students response for option b and c.

Research Focus

Using of non-routine problems in school mathematics may be beneficial to enhance creativity in problem posing. Non-routine problems are not solved through the use of algorithms. These problems have got openness, critical thinking and novelty features by heuristic approach(Schoenfeld, 2014).

Non-standardized word problems have different solving process in comparison to routine word problems. To solve non-standardized word problems, real-life knowledge have to be used in solving process. In other words, students must apply their making sense of answers.

Sense making of mathematical situations appeared in the relationship between conceptual knowledge and its interpretation(Resnick, 1988). Students failed to connect mathematical learning to real life. For these reason, DWR problems has been manifested as a useful tool for sense making mathematics. Prominent scientist as Silver, Li and Rodriguez has explored DWR problems on students.

METHODOLOGY

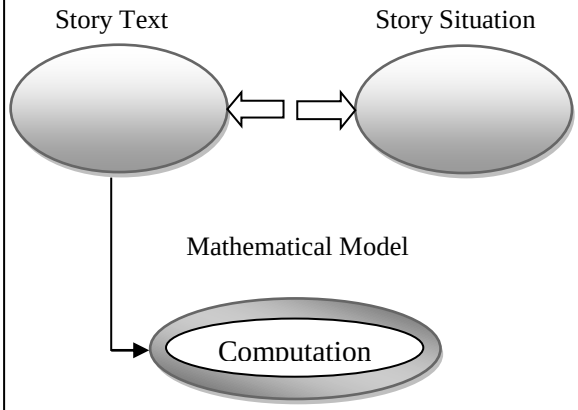
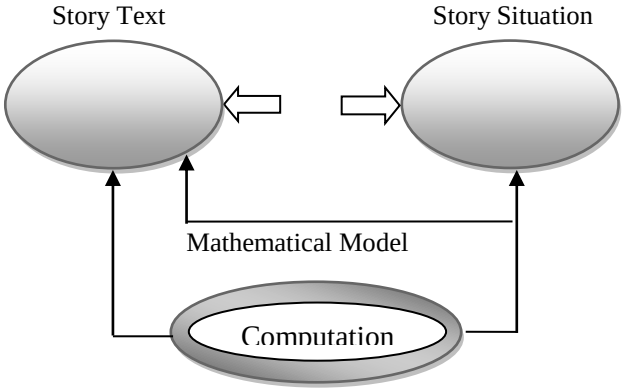
Students cannot integrate their experiences of real life to school mathematics or exact opposite in terms of realistic approach to problem solving. In other words, they cannot correlate real life and school mathematics(Reusser & Stebler, 1997). Because students think that problems have a standard solution process in virtue of mathematics textbook and questions which are used by mathematics teachers(Blum, 2015; Bonotto, 2005; Greer, 1997). For overcoming this problem, teachers should avoid stereotype questions and prompt their students to think critically and creatively on real life problems(Bonotto, 2005).

Silver (1994) emphasized that some problems could be solved correctly but it is not enough to ensure a successful solution for DWR problems. For instance, National Assessment of Educational Progress asked DWR questions: "An army bus holds 36 soldiers. If 1.128 soldiers are being bused to their training site, how many buses are needed?" Only 24% of students who were 13 years old was able to solve this problem correctly and gave answer as 32. In DWR problem, result of calculation of problem is not an integer and what is worse it has a remainder. In consideration of this, student who meets with DWR problem needs to interpret and link up real world. On that sense, DWR problem is one of the first and best types of making sense

problems(Chen, Van Dooren, & Verschaffel, 2011).

DWR problem is regarded as non- routine and complex problem. Although response to this problem type by arithmetic operation is right answer, it is senseless answer. As for mentioned example that solver may find answer is 31, 3. But this response ignores realistic thinking because of non-integer outcome. Or solver may find answer as 31. For this response it can be said that student reduce the reality looking at the literature, the first noticeable study belongs Silver, Shapiro and Deutsch in 1993. They explored a hypothesized that called semantic-processing model. Participants of this study were 200 middle school students. They solved an augmented quotient division with remainder problem(Arikan & Unal, 2013). They were asked interpretation. The model being developed by Silver (1994) was presented as following:

Table 1. The models for a solid explanation of students' failure to solve DWR problem

Schematic representation of hypothesized unsuccessful solution	Schematic representation of an idealized successful solution
	

It was found that these models enable to analysis students' failure in DWR problems. Also, the results showed that the students had difficulty in explanation of their mathematical thinking and reasoning.

Having said that Silver (1994) carried out a research on participants who had not treated division with remainder and were 3rd grade students. So, the participants did not use division solution but non-division strategies for DWR problems. The result of this study, it was stated that using non-division strategies for solving to DWR problem helps to make sense problem. The participant used additive approach or multiplicative approach for solving DWR problem. With the aim of this study, researchers emphasized that younger students succeed where older students fail in making sense of DWR problem.

In the study, DWR problem solving was used for comparison cross-national students' sense making. Although 89% of the US students and 94% of the Chinese students implemented correct execution. When it comes to interpretation, 42% of the US students and 36% of the Chinese students gave a correct answers.

Cooper and Harries (2003) studied on working class students who were 11-12 years old. The participants were asked DWR problem for solving and interpreting. For this reason, the participants were presented DWR problem at first step and immediately afterwards they responded four yes-no questions which comprised the DWR problem to obtain mounting evidence of students' interpretations. Furthermore, to identify the underlying results of students'

failure, researchers had an interview with them. According to the researchers, the result of this study was not a pretty sight as expected. The researchers remarked that these students, who were in working class, when they faced with DWR problems, they could not identify proper operations. They failed by using either multiplication or a division without rounding up.

Rodríguez et al. (2009) tried to find whether participants of this study made mistakes in initial representation or correct interpretation of numerical result. Accordingly, 25 students of 45 participants were asked to solve equal groups' problem and the rest of this participants were asked to solve comparison problems as following Table 2.

Table2. Type of Division Situation

Equal group problems		Comparison problems
RND	Jun's grandfather gives a box of 26 balloons to his 6 grandchildren to share, so that all have the same number of balloons. Have many balloons will each grandchild receive?	Luis has 25 crayons. He has approximately 3 times as many crayons as Juan. How many crayons des Juan have?
RD	The pizzeria in my neighborhood has prepared 21 kg of pizza dough. With all that dough, they made 6 medium pizzas. How much dough would they use to make each pizza?	Ana weighs 45 Kg. Her weight is 2 times as much as Pedro's. How much does Pedro weigh?
RR	My school is having a fun day, and the 35 eight-graders have had a mini-Olympics with 4 teams. If all the teams have the same number of students, how many teams could have reserves?	Maria bas 23 sweets. If we want her to have exactly 2 times as many sweets as Alicia, how many sweets will Maria have left over?
RQPI	This weekend, 19 friends are going to ride boats in the park. If there are only 4 boats, how many friends would have to get into each boat so that they could all ride at the same time?	Luis has read 14 books, nearly 5 times as many books as Ana How many books will Ana have she finishes the book she's reading now?
Note: RND-Remainder Not Divisible, RD-Remainder Divisible, RR-Remainder as Response, ROPI-Readjusted Quotient by Partial increments		

The result of this study, when students understood the problem, even if they found wrong numerical answer, they could interpreted properly. When students

Misunderstood or did not understand the problem, their interpretations focused on superficial information of the problem. Namely, the participant of this study had difficulties because of initial misunderstanding of the problems.

Chen et al. (2011) researched pre-service and in-service elementary school teachers' realistic approaches for division of remainder problems. The participants were subject to the task that consists of three DWR problem solving and three problem posing according to symbolic expressions. The examination of participants' endeavors showed that the teachers displayed realistic way of thinking both of problem solving and problem posing tasks. Also, the researchers examined evaluation of these participants to their pupils' responses for this type of questions. They found a correspondence between teachers' responses to DWR problems and their evaluation of pupils' replications.

The participants of this study were 133 seventh grade students and 117 eighth grade students. Each student was asked to pose a problem according to figural pattern and solve a DWR problem.

We wonder our students' problem posing abilities according to pattern and problem solving abilities of authentic word problem, DWR problem, which is used for meaning of concordance between in and out of school situations. In line with this purpose, two question of the task which had implemented by Cai (1998) was used for our study. The aim of our study was to compare both the results of Cai (1998) study's and two different grade students (seventh and eighth grade students) in Turkey. The study was non-experimental comparative study to describe current situation of seventh and eighth grade students in Turkey. The data were collected by paper-and-pencil exam.

When problem posing question was analyzed, the problems were classified as extension and non-extension. Extension and non-extension problem types were categorized as factual, comparative or rule-based. Students' responses were classified by content analysis.

DWR problem was inspected according to solution process, solution and interpretation of the numerical result. Students' responses were evaluated that if solution process, answer and interpretation are full complement, then student has got 3 points; if solution process and answer are full complement, then student has 2 points; if solution process only is full complement, then student has 1 point. According to this evaluation, the result of the examination was at Table 5. For comparing seventh and eighth students in DWR problem solving, independent-T-test was used. The study was carried out both of qualitative (content analysis) and quantitative (independent-t test) research.

RESULTS

For 7th grade students: Number of posed problems was 110 (mean number: 0,827). While 46 students of the participants could not succeed in problem posing task, 23 students remained unanswered. The distribution of posed problems was presented in table 3.

Table 3. Content Analysis of Problem Posing Task

Categories	Extension	Non-extension
Factual	53	25
Comparative	4	14
Rule-based	14	-

As seen in Table 3, 64, 54% of the problems was in extension type, 17, 29 of the students have unanswered and 34, 58% of the students generated a problem irrelevant to figural pattern.

Looking at the DWR problem, 109 students selected correct solution process. 2 students of them gave preference to subtraction but they could not make subtraction continue because of lasting long. 81 students of them accomplished at execution of division computation properly. When analyzed interpretation of numerical results for making sense, although 55 students of 81 found answer as 31, they misinterpreted like that "12 persons stand in bus", "12 persons do not get on the bus" and "12 persons are pressed with the others". Two students were abstain for comment. Namely, 14 students of the participants could solve DWR problem and interpret the result of operation rightly.

For 8th grade students

The number of posed problems was 140 (mean: 1.206). While 5 students of the participants could not pose a problem according to figural pattern, 23 students left empty this task. The distribution of posed problems was stated in Table 4.

Table4. Content Analysis of Problem Posing Task

Categories	Extension	Non-extension
Factual	87	5
Comparative	6	5
Rule-based	37	-

92, 85% of the problems was in extension type, 19, 65 of students' responses were empty and 4, 27% of student have not generate a problem as desired in Table 4.

DWR problem was analyzed according solution process, numerical result and interpretation. 99 students of 116 participants had correct solution process (one student of them selected subtraction but could not complete the process). While 26 students solved DWR problem accurately and interpret rightly, 16 Students grounded up their solution to 32 but there was absence of an interpretation. 27 students carried out solution process properly, but they expounded in a different way like that "answer is 12", "12 persons do not get on the bus", " answer is 310, so that 311 buses are needed", "12 persons stand in bus" and " 12 persons distribute 12 buses so 31 buses are needed. The quantitatively comparison of seventh and eighth grade students according to problem solving task

Table 5. Independent Samples Test

	Levens test for Equality of variances						
	F	Sig.	t	df	Sig. 2	Mean Deferent	Std. Error difference
Score Equal Variance Assumed	8.226	0.005	0.923	192	0.35	-0.81	0.088
Equal Variances not Assumed			0.927	187.52	0.35	-0.81	0.87

With reference to Table 5, there is no significant difference between seventh and eighth grade students for solving DWR problem.

Even though the number of eighth grade students who answered completely DWR problem more than seventh grade students, seventh grade students outnumber in terms of solution process and the numerical result. This means that seventh grade students have difficulties making sense of DWR problem

CONCLUSION

In the present study, our students were not successful as Chinese and U.S students. This situation may be explained that these students had not experienced problem posing activity. Even though problem posing is in Turkish curriculum, do teachers live problems in practice regarding to time or experience? Whether in-service training is needed regarding to problem posing studies might be carried out whether both pre-service and in-service teachers need training for problem posing.

Also, these students had not solved non-routine problems immensely. In fact, the more class grade increases the more students approach to non-routine word problems realistically. At this stage, students must be fostered by solving non-routine and realistic problems (for making sense) in school mathematics.

Martínez-Cruz and Contreras (2002) emphasized that although there were many study concerning to the problem posing, it could not be remarked as problem solving in terms of curriculum. It may be said that this issue still continue for Turkey since comparison of Cai (1998) study which was in 1998 and our study. Because 223 Chinese students posed 1588

problems such that problems per capita was 7.12 and US students posed 861 problems such that problems per capita was 4.76. As opposed to the Cai's study, Turkish students who selected randomly posed problems per capita less than US and Chinese students16. That is to say, we come from behind curriculum of US and Chinese almost 20 years. As mentioned before, Kilic (2013) stated in her study that problem posing takes place in Turkish Curriculum from 2006. At this point, a question comes to mind: even though problem posing is in Turkish curriculum, do teachers live problems in practice regarding to time or experience? In the study of Arikan and Unal (2013), second grade students' problem posing ability was examined. For supporting this study in many ways, an interview was carried out with the class teacher and the teacher asked whether in-service training is needed regarding to problem posing and she answered that studies of universities themselves should had shared with the class teachers relevant to problem posing. In other words, class teachers or mathematics teachers may be needed to train for problem posing in Turkey. Hence, studies might be carried out whether both pre-service and in-service teachers need training for problem posing.

When students meet word problem which requires arithmetic operations, they do not consider realistic conditions. This issue originated from solving stereotype mathematics problems in school life. Students may not come across non-routine problems. On the contrary, they may obliged to solve non-routine problems like DWR problems. In this situation, problem solving by itself has no mean and result of the solution of the problem must be made sense at the same time. Seventh and eighth grade students have no significant difference in solving DWR problem consistent with Table 5. While students who conducted solution process rightly but performed adversely in making sense of the answer are 67 of seventh grade students and 55 of eighth grade students. In Cai's study 3.31%of the Chinese and 38% of the US students operated correctly computational process and appropriate interpretation for their responses. However, 10.52% of seventh grade and 22.22% of eighth grade students executed suitable both of computation and interpretation in the sense of realistic in our study. Unfortunately, our students have no éclat as compared with US and Chinese students. At this stage, students must be fostered by solving non-routine and realistic problems (for making sense) in school mathematics. At this juncture, Silver (1994) stated "students' performance may have been adversely affected by the dissociation of sense making from the solution of school mathematics problems points to the need for more instructional attention to sense making as a part of school mathematics instruction. To illustrate this suggestion, this problem can be written according to figural pattern problem posing task:

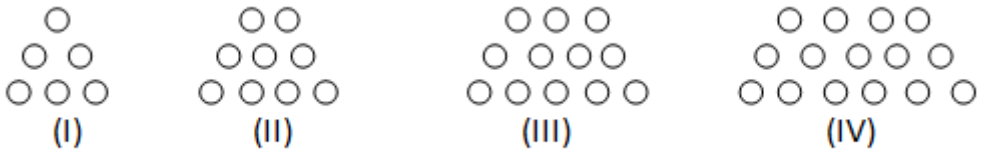


Figure 1. Pattern for problem posing

"Ayşe has got 44 marble. Ayşe wants to form the pattern in line with above mentioned figure. How many marbles does she need?" Students' realistic approaches can be observed on their mathematical problem posing skills alongside problem solving.

REFERENCES

Arikan, EE, & Unal, H. (2013). The Analysis of Mathematical Problem Posing Skill of Elementary Second Grade Students. *Amasya Educational Journal, 2*(2), 305-325.

Blum, Werner. (2015). *Quality Teaching of Mathematical Modelling: What Do We Know, What Can We Do?* Paper presented at the The proceedings of the 12th international congress on mathematical education.

Bonotto, Cinzia. (2005). How informal out-of-school mathematics can help students make sense of formal in-school mathematics: The case of multiplying by decimal numbers. *Mathematical thinking and learning, 7*(4), 313-344.

Cai, Jinfa. (1998). An investigation of US and Chinese students' mathematical problem posing and problem solving. *Mathematics Education Research Journal, 10*(1), 37-50.

Chen, Limin, Van Dooren, Wim, & Verschaffel, Lieven. (2011). An investigation on Chinese teachers' realistic problem solving abilities and beliefs. *Journal of Mathematics Education, 4*(2), 80-96.

Cooper, Barry, & Harries, Tony. (2003). Children's use of realistic considerations in problem solving: Some English evidence. *The Journal of mathematical behavior, 22*(4), 449-463.

Ellerton, Nerida F. (1986). Children's made-up mathematics problems—a new perspective on talented mathematicians. *Educational Studies in Mathematics, 17*(3), 261-271.

Greer, Brian. (1997). Modelling reality in mathematics classrooms: The case of word problems. *Learning and instruction, 7*(4), 293-307.

Kilic, Cigdem. (2013). Turkish primary school teachers' opinions about problem posing applications: Students, the mathematics curriculum and mathematics textbooks. *Australian Journal of Teacher Education (Online), 38*(5), 144.

Krutetskiĭ, Vadim Andreevich, WIRSZUP, Izaak, & Kilpatrick, Jeremy. (1976). *The psychology of mathematical abilities in schoolchildren*: University of Chicago Press.

Lai, Yinghui, Zhu, Xiaoshuang, Chen, Yinghe, & Li, Yanjun. (2015). Effects of mathematics anxiety and mathematical metacognition on word problem solving in children with and without mathematical learning difficulties. *PloS one, 10*(6), e0130570.

Martínez-Cruz, Armando M, & Contreras, Jose N. (2002). Changing the goal: An adventure in problem solving, problem posing, and symbolic meaning with a TI-92. *The Mathematics Teacher, 95*(8), 592-597.

Mathematics, National Council of Teachers of. (2000). *Principles and standards for school mathematics* (Vol. 1): National Council of Teachers of.

Resnick, Lauren B. (1988). Treating mathematics as an ill-structured discipline. Washington, DC: Office of Educational Research and Improvement (ED),.

Reusser, Kurt, & Stebler, Rita. (1997). Every word problem has a solution—The social rationality of mathematical modeling in schools. *Learning and instruction, 7*(4), 309-327.

Rodríguez, Purificación, Lago, M Oliva, Hernández, M Lourdes, Jiménez, Laura, Guerrero, Silvia, & Caballero, Sonia. (2009). How do secondary students approach different types of division with remainder situations? Some evidence from Spain. *European journal of psychology of education, 24*(4), 529.

Schoenfeld, Alan H. (2014). *Mathematical problem solving*: Elsevier.

Silver, Edward A. (1994). On mathematical problem posing. *For the learning of mathematics, 14*(1), 19-28.

Silver, Edward A, & Cai, Jinfa. (1996). An analysis of arithmetic problem posing by middle school students. *Journal for research in mathematics education, 27*(5), 521-539.

Singer, Florence Mihaela, Ellerton, Nerida F, & Cai, Jinfa. (2015). *Mathematical problem posing*: Springer.

Watson, Anne, & Mason, John. (2002). Student-generated examples in the learning of mathematics. *Canadian Journal of Math, Science & Technology Education, 2*(2), 237-249.