



## **A Comparative Study of the Accuracy of Fuzzy AHP and Classical AHP for Effective Indices Ranking on Products Prioritizing Case Study of Fars Green Pipes Factory**

**Abbas Talebbeydokhti\* and Razieh Mardani**

Department of Industrial Engineering, Najaf Abad Branch, Islamic Azad University, Isfahan, Iran

\*Corresponding Author: talebbeydokhti@yahoo.com

**Abstract:** In today's real world, many quantitative and qualitative factors such as profitability, quality, price, flexibility, together with developments and innovations must be taken into account to make decisions. For this purpose, to determine the rates and the weights, the linguistic variables can be used and they can be expressed in the form of fuzzy numbers. In this study, one of the multi-criteria decision-making methods called Analytic Hierarchy Process (AHP) in the states of Classic and Fuzzy are investigated to obtain the weight of effective parameters on products prioritizing of Fars Green Pipes Factory. Following data collection, the two mentioned methods are used comparatively. Due to the differences in the obtained results, the accuracy is calculated by the index of Mean Square Error (MSE) and the following results are obtained: 1. in both methods, the profitability index and the ease of transportation have respectively the highest and the lowest weight of importance at the related case study .2. In conformity with the existent reality, the fuzzy method has higher accuracy than classical method. This difference is confirmed using the index of mean square error .

**Keywords:** Ranking, Products Prioritizing Indices, Multi Criteria Decision Making, Analytical Hierarchy Process, Index of Mean Square Error

### **INTRODUCTION**

Each of managers' tasks, such as planning, organizing, and controlling is the manifestation of decision-making. The effectiveness of the decision, which is a function of different quantitative and subjective factors, shows that the managers have done their tasks effectively and efficiently and ultimately productivity that is the major factor of capability in any organization to achieve its strategic goals in long term will be fulfilled. Therefore, it is natural when managers face large and complex factors; this process actually works difficultly and slowly .

As a result, the need to concentration, serious discussion and achieving general agreements as well as the use of appropriate decision-making tools prevent the disorder in decision-making as possible. Among the most important decisions of directors are identification and ranking of effective indices on products prioritizing based on achieving an appropriate number of desired goals? In this regard, identifying influential indices on products prioritizing and obtaining

## **A Comparative Study of the Accuracy of Fuzzy ...**

the importance level of these criteria in business interests achievement seems to be imperative.

Regard to the fact that ambiguity and uncertainty are inherently dominant in the management sciences, particularly in the areas of planning and decision-making<sup>1</sup>. Important applied topics are associated with linguistic propositions. Fuzzy sets, compared to the classical sets, are one of the most efficient methods in decision-making issues<sup>2</sup>. The input and output data of ambiguous topics are examined by imprecise measures values. In recent years many efforts have been made to resolve these ambiguities and uncertainties that ultimately have led to the application of fuzzy sets theory in multi-criteria evaluation methods<sup>3</sup>. Fuzzy sets theory published by Professor Lotfi Zadeh in 1965 indicates the human behavior understanding. It is applied to analyze the issues that are complex and ambiguous and analyzing them by traditional common methods is faced with high sensitivity. This theory allows the user to do verbal judgments by linguistic expressions.

Generally, linguistic descriptions in comparison to numeric descriptions, are a bit more specific and more accurate<sup>4</sup>. Fuzzy methods compared with classical methods have greater ability to measure and predict exactly. One of the classical AHP problems that often cause concern to decision makers is the subjective judgments in formulating paired comparison matrix. Therefore, classical Analytic Hierarchy Process is not efficient for ranking the options based on paired comparisons<sup>5</sup>. In other words, this method does not have the capability to reflect the human thinking style completely<sup>6</sup> and it is better predict for long-term and make decisions in the real world using fuzzy sets<sup>7</sup>. When uncertainty must be observed in some or all of the paired comparison values, in the classical AHP method random values are used that are actually not appropriate.

It is more realistic that a decision maker can replace fuzzy judgments with vague and imprecise judgments<sup>8</sup>. The results of fuzzy AHP are more documented and are closer to reality<sup>9</sup>. In fact, the fuzzy decision theory tries to resolve ambiguity and uncertainty inherent in the preferences, goals and limitations on the issues of decision-making<sup>10</sup>. According to this claim, the present study aimed to evaluate the accuracy of the fuzzy Analytic Hierarchy Process (FAHP) and classical Analytic Hierarchy Process (AHP) for ranking the effective indicators on products prioritizing of Fars Green Pipes Factory by proposing the following questions :

- 1-** What are the effective indicators on products prioritizing ?
- 2-** Is there any difference between fuzzy Analytic Hierarchy Process (FAHP) and classical Analytic Hierarchy Process (AHP) in the evaluation and determination of the indicators importance weight ?
- 3-** Which of AHP and FAHP methods has higher accuracy?

## **LITERATURE REVIEW**

This section presents a summary of the studies done in the field of AHP and FAHP. Extensive research has been done in the field of research methods some of which is referenced in the following :

F.T.S Chan and K.H Chan have created a model for supplier selection and have examined its application in the advanced technology industries. They used Analytic Hierarchy Process and principle of quality management system to build the supplier selection model. Regard to the fact that not all criteria of the issue for supplier selection is quantitative, only a few have been used to formulate the optimization<sup>11</sup>.

Jabbari has ranked Iranian industries based on economic indicators. In this regard, he considered nine economic indicators, including value added and production value as the benchmark and Iranian industries as the option. Using AHP method, he attempted to rank them.

Results of multivariate analysis allocated the first rank to the production of basic chemicals<sup>12</sup>.

Chi and Wu conducted a study on the financial assessment. In the first stage using analytic hierarchy process, they evaluated and adjusted the calibration of the initial financial evaluation system. They investigated thirteen financial evaluation indicators and categorized them into 14 groups. In the second stage using the DEA, they introduced a model whose output specified units that are more efficient<sup>13</sup>.

Ghodsi Poor presented a model to make decision for supplier selection using analytic hierarchy process and linear programming<sup>14</sup>.

Chan and Kumar introduced a model using fuzzy analytic hierarchy process for the organization supplier's selection with regard to the risk factor<sup>15</sup>.

Chou and Liang combined fuzzy sets theory, the hierarchical analysis and entropy concepts and used the presented model in performance evaluation and ranking shipping companies<sup>16</sup>.

Duran and Aguilo introduced a hierarchical approach based on fuzzy multi-criteria methods for the evaluation and justification of advanced manufacturing technologies and involved strategic and economic criteria in the model<sup>17</sup>.

Tolga et al., investigated the issue of operating system selection using fuzzy replacement analysis and fuzzy analytic hierarchy process. In this study, the economic and non-economic aspects of the selection and ranking of operational systems were considered<sup>18</sup>.

## **MATEREALS AND METHOD**

One of the important steps before evaluating the study subject is to select its appropriate model or models .

The model examined in this study consists of four stages. In the first stage, the influencing indicators on products prioritizing of the factory are identified. In the second stage, using paired comparison matrix and classical analytic hierarchy process method the weight of each factor is set. In the third stage, the weight of indexes is obtained using fuzzy analytic hierarchy process. Finally, in the fourth

## A Comparative Study of the Accuracy of Fuzzy ...

stage the error rate of each of the AHP and FAHP methods is calculated by computing the mean square error (MSE). Implementing these four steps is described below :

### The first stage (indexes selection) :

One of the main components and stages of this study is deciding on the indexes selection. Choosing the best set of indexes is a critical stage in any decision-making study. The several effective indicators on products prioritizing are identified through library and internet study of applied research and invoking productivity technical resources, production management, production scheduling, and so on. Then, 10 factors that are the most influential on factory products prioritizing have been identified through interviews with the statistical population, which included four specialists and experts in the production of Shiraz green pipes (Lule sabz Gostar) factory.

These indicators include profitability, ease of production, product quality, product safety, time consuming to produce, market availability, the environmental impact of production, associating with technological developments, customer demand, and ease of transportation .

### The second stage (classical analytic hierarchy process) :

At this stage, through Analytic Hierarchy Process model the problem is analyzed, and is divided into several easier parts. After determining the indicators, paired comparisons are done between indices. Then the following algorithm is gone by:

1. Filling the paired comparisons matrix: to fill the matrix of paired comparisons, the scales of 1 to 9 is used to determine the relative importance of each indicator compared to the other indicators (Table 1).

**Table 1.** Paired comparison in classical Analytical Hierarchy Process <sup>19</sup>

Importance degree in paired comparisons	Numerical value
Equal value	1
Slightly higher value	3
More value	5
Much more value	7
Absolute value	9
Intermediate value	2,4,6,8

Note: when the elements  $i$  and  $j$  are compared, one of the numbers in the above table is assigned to it. In comparison of the element  $j$  with  $i$ , the inverse of that number is assigned. So that we always have:

$$X_{ji} = 1/x_{ij}$$

In addition, in paired comparisons matrix, the row  $i$  is compared with the column  $j$ . therefore, all the elements of the main diameter of the matrix is the number one. Furthermore, any value below the main diameter is the inverse of the value above the diameter.

2. Obtaining the arithmetic mean of each matrix row to the normalized of paired comparisons (which refers to the relative weight): In this method, first each column is normalized and then the elements line mean is computed to obtain the weight vector.

3. To calculate the adjustment vector: the elements of weight set vector are divided by the relative weights vector. Resulting vector is called "inconsistency vector".

4. Computing the largest eigenvalues of the paired comparisons matrix: To compute the largest eigenvalues of the paired comparisons matrix, the elements mean of adjustment vector is calculated.

5. Calculation of inconsistency index: inconsistency index is calculated as follows:

$$II = (\lambda_{\text{Max}} - n) / (n - 1)$$

6. Calculation of inconsistency rate  
Inconsistency rate is calculated as follows:

$$IR = II / IRI$$

Where, random inconsistency index is a value that is extracted from the corresponding table. Random inconsistency index table is obtained based on simulations as follows (Table 2).

In order to assess the validity of the questionnaire from the apparent validity, using the professors and experts' opinion, the research questionnaire is revised and final set. In order to assess reliability of the questionnaire, test-retest method is used.

Therefore, the research questionnaire is first distributed among 15 directors and senior experts of staff domain involved in evaluating the performance of Hormozgan University of Medical Sciences. The questionnaire reliability is computed using Cronbach's alpha coefficient of 0.87. Then, the questionnaire is conducted among the entire population of the survey that is equal to 0.92. The reliability of each of the aspects of the Balanced Scorecard is as the table below shows.

The analysis is performed using descriptive statistics and simple frequency and relative frequency. It is conducted based on fuzzy logic and MATLAB analysis software.

**Table 2.** random inconsistency index<sup>20</sup>

n	3	4	5	6	7	8	9	10
IRI	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.51

If the inconsistency rate is less than or equal to 1 ( $IRI \leq 1$ ), there is consistency in paired comparisons and it can be continued. Otherwise, the decision maker must revise in the paired comparisons .

The third stage (fuzzy analytic hierarchy process):

**Step 1:** Drawing hierarchical chart

## A Comparative Study of the Accuracy of Fuzzy ...

**Step 2:** Defining the fuzzy numbers in order to perform paired comparisons: in the present study verbal linguistic has been used instead of absolute numbers to perform paired comparisons and to determine the indexes weight. Table 3 presents the verbal statements to describe the criteria importance compared to each other.

**Table 3.** Linguistic phrases to describe the importance of standards <sup>21</sup>

Fuzzy number	Linguistic variable	Fuzzy number scale
1	Equal	(1,1,1)
2	Infinitely less important	(1,2,3)
3	Very less important	(2,3,4)
4	Less important	(3,4,5)
5	Medium	(4,5,6)
6	Important	(5,6,7)
7	Very important	(6,7,8)
8	Extremely important	(7,8,9)
9	Absolute importance	(8,9,10)

For fuzzy expressing of the problem and changing linguistic variables to fuzzy numbers, the triangular fuzzy numbers are used in the form of  $\tilde{A} = (l, m, n)$ . In this study, the membership function selected for fuzzy numbers is as follows:

$$\mu(x) = \begin{cases} 0 & x < l \\ x - l / m - l & l \leq x \leq m \\ u - x / u - m & m \leq x \leq u \\ 0 & x > u \end{cases}$$

**Step 3:** Formulation of paired comparison matrix ( $\tilde{A}$ ) using fuzzy numbers

$$\tilde{A} = \begin{bmatrix} \tilde{1} & \dots & \tilde{a}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{a}_{n1} & \dots & \tilde{1} \end{bmatrix} \text{ s.t. } \tilde{a}_{ij} = \begin{cases} 1 & i = j \\ \tilde{2}, \tilde{3}, \dots, \tilde{9}, \tilde{2}^{-1}, \tilde{3}^{-1}, \dots, \tilde{9}^{-1} & i \neq j \end{cases}$$

**Step 4:** Calculation of  $S_k$  for each of the rows.  $S_k$  which is a triangular fuzzy number is calculated from the following equation:

$$S_k = \sum_{j=1}^n M_{kj} \times \left[ \sum_{i=1}^m \sum_{j=1}^n M_{ij} \right]$$

Where  $k$  denotes the number of rows, and  $i$  and  $j$  respectively indicate the options and indexes.

**Step 5:** Calculation of largeness degree of  $S_k$ s compared to each other. If it is assumed that  $M_1$  and  $M_2$  are two triangular fuzzy numbers, as follows:

$$M_1 = (l_1, m_1, u_1) \quad , \quad M_2 = (l_2, m_2, u_2)$$

Then their largeness degree is calculated as follows:

$$V(M_1 \geq M_2) = \begin{cases} 1 & \text{if } m_1 \geq m_2 \\ 0 & \text{if } l_2 \geq l_1 \\ \frac{u_1 - l_2}{(u_1 - l_2) + (m_2 - m_1)} & \text{otherwise} \end{cases}$$

**Step 6:** Calculation of the criteria weights in paired comparison matrices. To achieve this goal the following equation is used:

$$w(i) = \text{Min}\{V(S_i \geq S_k)\} \quad k=1,2,3,\dots,n \quad k \neq i$$

Normalizing the values obtained above, through the following formula, the weight vector of each criterion is obtained:

$$W_{(i)} = W'(i) / \sum_{j=1}^n W'(j)$$

**Fourth stage (calculation of Mean Square Error):**

In this index, the smaller errors will be weighted larger. In other words, in the comparison of methods, the method that has a much smaller error is elected. This means that the method has higher accuracy. In the present study to compare the accuracy and closeness to the existent reality in both classical and fuzzy analytic hierarchy processes, MSE (Mean Square Error) is used. To compute Mean Square Error,  $r_i$  is the coefficient obtained from fuzzy or classical method,  $k_i$  is the coefficient obtained from the professionals viewpoint for each index and  $n$  is the index number. The mathematical equation of this index is as follows<sup>22</sup>:

$$MSE = \frac{\sum_{i=1}^n (r_i - k_i)^2}{n}$$

**RESULTS**

The results obtained from the classical analytic hierarchy process (AHP) shows that the profitability index has the first rank, and the transport index has last rank. Weights obtained by AHP method for each of the indicators is given in Table 4. Consistency rate in this method is equal to 0.812, which shows that the performed calculations and the ranking results have high accuracy and the consistency between the criteria is met.

**Table 4.** Weight of indexes by AHP method

Transportation	Raw materials	Technology developments	Environment	Market	Time consuming	Safety	Production capacity	Ease of production	Profitability	Standard
10	3	2	8	7	9	5	6	4	1	Rank
0.001	0.15	0.151	0.082	0.08	0.03	0.1	0.08	0.131	0.205	Normalized weight

## A Comparative Study of the Accuracy of Fuzzy ...

The results obtained from the fuzzy analytic hierarchy process (AHP) shows that the profitability index has the highest rank, and the transport index has lowest rank of importance. Weights obtained by FAHP method is given in Table 5. Consistency rate in this method is equal to 0.868, which shows that the performed calculations and the ranking results have high accuracy. It means that the consistency between the criteria is met in the best way.

**Table 5.** Weight of indexes by FAHP method

Transportation	Raw materials	Technology developments	Environment	Market	Time consuming	Safety	Production capacity	Ease of production	Profitability	Standard
01	2	9	7	8	6	3	5	4	1	Rank
0	0.22	0.01	0.06	0.0	0.1	0.1	0.11	0.12	0.25	Normalized weight
				1		3				t

Comparison of two methods of fuzzy and classical with the existent reality of the effective parameters on products prioritizing (the normalized values obtained from the responses of experts) shows that the results of indexes ranking in fuzzy AHP method, in terms of priorities and accuracy, have very high consistency with the results obtained from the experts' point of view. However, the results of classical analytic hierarchy process, despite of weaker accuracy and compliance, is different in terms of priority order. Another sign of fuzzy AHP greater precision compared to the classical method is the relative closeness of indexes and their higher consistency with each other, which is clearly visible in this study. So that the dispersion coefficient in fuzzy method is 0.045 and in classical method is 0.203. According to the results of Mean Square Error (MSE), the interval between the indexes coefficients of classical method and fuzzy method is calculated by the normalized ratio derived from experts' opinion respectively 0.03,161 and 0.00,109.

## CONCLUSIONS

Evaluation of effective parameters on products prioritizing, particularly, determination of the importance weight of each of them play a constructing role in the development and improvement of the manufacturing firms' status. In this regard, two decision-making methods of classical and fuzzy are used. Although in both classical and fuzzy Analytic Hierarchy methods the profitability index and the ease of transportation have respectively the highest and the lowest weight of importance, the indexes ranking results in fuzzy method, in terms of priorities and accuracy, are very highly consistent with reality. In MSE index the interval between the indexes coefficients of classical method and fuzzy method is

calculated by the normalized ratio derived from experts' opinion respectively 0.03,161 and 0.00,109. MSE value is much smaller than in the fuzzy method and it can confirm the above claim.

## REFERENCES

1. Amini Faskhudi, A. (2005) .the application of fuzzy logic inference in planning and regional development studies, *Journal of Knowledge and Development*;17 :45 - 55.
2. Zadeh, L.A. (1965). Fuzzy sets. *Information and Control*; 8: 231-252.
3. Hwang, C.L. & Chen, S.J. (1992). *Fuzzy Multiple Attribute Decision Making Methods and Applications*, Springer, Berlin.
4. Zadeh, L.A. (1973). Outline of a new approach to the analysis of complex systems and decision processes, *IEEE Trans .System, Man, Cybernetics, and SMC-3*.
5. Sadeghi Sadegh Abadi, Z., Saber, S., Mehrabian, S. & Khodadoust, A. (2009). Combination of fuzzy AHP and fuzzy DEA for ranking decision making units, *Journal of Applied Mathematics, Lahijan Branch*; 6(34): 20 to 59.
6. Chang, D.Y. (1996). Application of the extent analysis method on fuzzy AHP, *European Journal of operational research*; 95:37-50.
7. Mohammadi Bolbolan Abad, S. & Iranmanesh, M. (2009). Selection and Management of Construction Holding Portfolio of Iran, *Journal of Industrial Administration, Sanandaj University*; 4(7):14-25 .
8. Khorshid, S. & Ghane, H. (2009). Ranking banking challenges using fuzzy AHP, *Journal of Industrial Management, Islamic Azad University of Sanandaj*; 4(9): 23-36.
9. Shishebori, D. & Hejazi, S.R. (2009). Applying fuzzy hierarchy process technique with the aim of selecting the most effective way of improving productivity, *Journal of Industrial Engineering*; 43(1): 9-17.
10. Zimmermann, H.J. (1996). *Fuzzy set theory and its application*, Kluwer Academic Publishers, Boston.
11. Chan, F.T.S., Chan, K.H. (2004). Development of the supplier selection model – A case study in the advanced technology industry, *European Journal of operational research*;156:56-67.
12. Jabbari, F. (2005). Iranian industries ranking based on economic selected indicators by AHP method, master's thesis of Industrial Engineering, Faculty of Engineering, and Tehran University.
13. CAI, Y. & Wu, W. (2007). Synthetic financial evaluation by a method of combining DEA whit AHP, *International Transactions in operational research*; 8:603-609.
14. Ghodsi Poor, H. (2008). *Analytical Hierarchy Process (AHP)*, Amirkabir University publication.
15. Chan, F.T.S., Kumar, N., Tiwari, M.K., Lau, H.C.W., Choy, K.L. (2012). Global supplier selection: A Fuzzy AHP approach. *International Journal of production research*; 140:144-149.

## **A Comparative Study of the Accuracy of Fuzzy ...**

16. Chou, T.Y., Liang, G.S. (2012). Application of fuzzy multi criteria decision making model for shipping company performance evaluation, *Maritime policy and management*; 28:375-392.
17. Duran, O. & Aguilo, J.(2008). Computer-Aided machine -tool selection based on a fuzzy AHP approach , *Expert system with application* ; 34:1787-1794.
18. Tolga, E., Demircan, M. & Kraman, C. (2011). Operating system selection using fuzzy replacement analysis and analytic hierarchy process, *International journal of production economics*; 97:89-117.
19. Shakeri, A. & Salimi, F. (2006). Factors affecting investment attraction in Chabahar and prioritizing them using AHP mathematical technique, *Journal of Economic Research*; 20: 9-23
20. Taghipur Javi, A. (2009). Employment development feasibility with emphasis on changing industries, master's thesis in Geography and Rural Planning, Department of Geography, University of Sistan and Baluchestan.
21. Lee, A.H.I., Chen, W.C., Chen, C.J. (2008). A fuzzy AHP and BSC approach for evaluating performance of IT department in the manufacturing industry in Taiwan , *Expert Systems with Applications* ; 34:121-143.
22. Sarvari, A. A., Sadrolashrafi, S.A., Daneshvar Kakhaki, M. & Hatef, H. (2007). Effects of milk price changes on the welfare of producers and their predictability, *Journal of Agricultural Economics*; (1) 13-25.